Weight Minimization of Orthotropic Flat Panels Subjected to a Flutter Speed Constraint

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This paper deals with the minimum weight design of orthotropic panels subjected to a supersonic flutter speed constraint and to a system of uniform in-plane loadings. In approaching the problem, use is made of the methods of optimal control theory of distributed parameter systems. This leads to a set of necessary optimality conditions that, together with a supplementary condition ensuring that the flutter speed of the optimal panels coincides with the constrained one, constitute the governing optimality equations of the problem. An alternative form of the optimality equations is derived and a symmetry property of the optimal thickness distribution is placed in evidence. Numerical solutions are obtained via Galerkin's procedure, providing rough estimates of the optimal panel design. The results also show the influence of some important parameters, such as orthotropy ratio, in-plane loading, aspect ratio, and support conditions.

Nomenclature

a	= streamwise dimension of the panel
A	= structural nondimensional parameter, Eq. (1)
b	= spanwise dimension of the panel
D_{ij}	= panel bending rigidities, Eq. (2)
D_0	= bending rigidity of the reference panel
E_i	= nondimensional Young's moduli
G_{12}	= in-plane shear modulus of the orthotropic material, Eq. (2a)
h	= nonuniform panel thickness
h_0	= uniform thickness of the reference panel
J	= performance index, Eq. (6)
M_∞	= freestream Mach number
N_1,N_2	= in-plane uniform loadings
p_∞	= freestream gas pressure
t	= nondimensional thickness distribution
T_{11},T_{22}	= nondimensional in-plane loadings
$\hat{w}$	= transverse panel deflection
$\bar{w}$	= nondimensional transverse deflection
x_1,x_2	= nondimensional streamwise and spanwise coordinates
ϕ	= inverse aspect ratio
κ	= polytropic gas coefficient
Λ	= speed parameter
ν_i	= Poisson's ratios for the orthotropic panel
ω	= circular frequency of vibration
$\tilde{\omega}^2$	= frequency parameter
ω_0^2	= reference frequency
ρ	= mass density of the panel
τ	= time

Subscripts

$(\ )_{,x_i}$	$\equiv \partial(\ )/\partial x_i$
$(\ )_{,i}$	$\equiv \partial(\ )/\partial \bar{x}_i$
$(\ )_{,\tau}$	$\equiv \partial(\ )/\partial \tau$

Introduction

THE problem of the weight minimization of continuous structural members subjected to dynamic constraints constitutes a topic of major interest in all fields of engineering design. In this context, the minimum weight design of thin panels exposed over the upper face to a supersonic flow and satisfying a flutter speed constraint is of a great practical concern in the aerospace industry. Such an optimal design implies the determination of the thickness distribution minimizing the panel weight, while maintaining the same flutter speed as that of a uniform thickness reference panel. It appears from survey papers of this field[1,2] that most of the investigations focused on one-dimensional types of panels. In contrast, the aeroelastic optimization of two-dimensional panels—which poses considerable mathematical difficulties—has been investigated much less. As assessed in the field literature,[2] the first formulation of a two-dimensional panel flutter optimization problem (based on a simplified structural operator) was presented in Ref. 3. The pertinent optimality equations developed in Ref. 3 have been solved using a gradient projection algorithm by Pierson and Genalo[4] for the case of an isotropic panel without in-plane loadings. An approximate solution to the same problem was given by Van Keuren and Eastep[5] using a Galerkin procedure. A more general framework to this problem was developed in Ref. 6 by considering an orthotropic rectangular panel subject to tensile/compressive in-plane loadings and various boundary conditions.

The purpose of the present paper is to examine the weight minimization problem of flat two-dimensional orthotropic thin panels placed in a high supersonic flowfield and satisfying a flutter speed constraint. The obtained necessary optimality equations of the problem are supplemented by an integral condition due to Plaut,[7] ensuring that the flutter speed of the optimal panel coincides with that of the reference panel. Numerical solutions are obtained and the influence of the various parameters (viz., aspect ratio, orthotropicity degree, in-plane loadings) and of the support conditions on the resultings mass saving is put into evidence. For the sake of completeness, it should be mentioned that the flutter of nonoptimal orthotropic panels has been studied in a number of papers and reports (for an exhaustive bibliography on this subject, see Chap. 1 of Ref. 8).

Aeroelastic Governing Equations

Consider the case of a rectangular homogeneous and orthotropic flat panel of variable thickness $h(x_1,x_2)$, exposed to a high supersonic flowfield over its upper face and subjected to uniform in-plane loadings N_1 and N_2 (positive in compression), as shown in Fig. 1. In order to derive the pertinent aeroelastic equilibrium equations, it is assumed that: 1) the principal orthotropy axes coincide with the geo-

Received Feb. 13, 1985; revision received Sept. 16, 1985.

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metrical axes x_1x_2; 2) the panel is thin so that the classical bending theory of flat panels is considered valid; 3) the free-stream Mach number is sufficiently high so that linear piston theory aerodynamics is applicable; and 4) the panel is unbuckled under the compressive in-plane loadings. Under these assumptions and by disregarding the influence of structural and aerodynamic damping, the equation governing the aeroelastic equilibrium reads

$$[D_{11}(\hat{w},_{x_1x_1}+\nu_2\hat{w},_{x_2x_2}],_{x_1x_1}+[D_{22}(\nu_1\hat{w},_{x_1x_1}+\hat{w},_{x_2x_2}],_{x_2x_2}$$
$$+2A(D_{11}\hat{w},_{x_1x_2}),_{x_1x_2}+N_1\hat{w},_{x_1x_1}+N_2\hat{w},_{x_2x_2}-\rho h\hat{w},_{\tau\tau}$$
$$-\kappa p_\infty M_\infty\hat{w},_{x_1}=0 \tag{1}$$

where $A\equiv K[1-(\nu_1\nu_2)^{1/2}](E_2/E_1)^{1/2}$ is a structural nondimensional parameter and

$$D_{ii}\equiv D_{ii}(x_1,x_2)=E_i\frac{h^3(x_1,x_2)}{12(1-\nu_1\nu_2)} \qquad (i=1,2)$$

$$D_{12}\equiv D_{12}(x_1,x_2)=G_{12}h^3(x_1,x_2)/12 \tag{2}$$

denote the bending rigidities of the orthotropic panel, while

$$G_{12}=\frac{K(E_1E_2)^{1/2}}{2[1+(\nu_1\nu_2)^{1/2}]}$$

is the in-plane shearing modulus defined in terms of the nondimensional parameter $K(=1$ for isotropic material). E_i and ν_i are the Young's moduli and Poisson's ratios in the direction of the coordinate x_i $(i=1,2)$, respectively.

By defining the dimensionless variables $\bar{x}_1\equiv x_1/a$, $\bar{x}_2\equiv x_2/b$, and $\bar{w}\equiv w/a$, and by assuming harmonic panel motions $\hat{w}(x_1,x_2,\tau)=w(x_1,x_2)\exp(j\omega\tau)$, $j=(-1)^{1/2}$, the governing equation (1) can be rewritten as

$$\bar{E}_1[t^3(\bar{w},_{11}+\nu_2\phi^2\bar{w},_{22})],_{11}+\phi^2\bar{E}_2[t^3(\nu_1\bar{w},_{11}+\phi^2\bar{w},_{22})],_{22}$$
$$+2\bar{E}_1A\phi^2(t^3\bar{w},_{12}),_{12}+T_{11}\bar{w},_{11}+\phi^2T_{22}\bar{w},_{22}+\Lambda\bar{w},_1-\bar{\omega}^2t\bar{w}=0 \tag{3}$$

The quantities appearing in Eq. (3) are defined as:

1) Nondimensional Young's moduli (E_{ref} is a conveniently chosen reference modulus)

$$\bar{E}_i\equiv E_i/E_{\text{ref}} \qquad (i=1,2)$$

2) Nondimensional thickness distribution ($t=1$ for the reference panel)

$$t(x_1x_2)\equiv h(x_1,x_2)/h_0$$

3) Velocity parameter

$$\Lambda\equiv\kappa p_\infty M_\infty a^3/D_0$$

4) Frequency parameter $(1/\omega_0^2\equiv a^4h_0/\pi^4D_0)$

$$\bar{\omega}^2\equiv\pi^4(\omega^2/\omega_0^2)$$

5) Nondimensional in-plane loading parameters

$$T_{ii}\equiv N_ia^2/D_0 \qquad (i=1,2)$$

6) Inverse aspect ratio

$$\phi\equiv a/b$$

7) Reference bending rigidity

$$D_0\equiv\frac{E_{\text{ref}}h_0^3}{12(1-\nu_1\nu_2)}$$

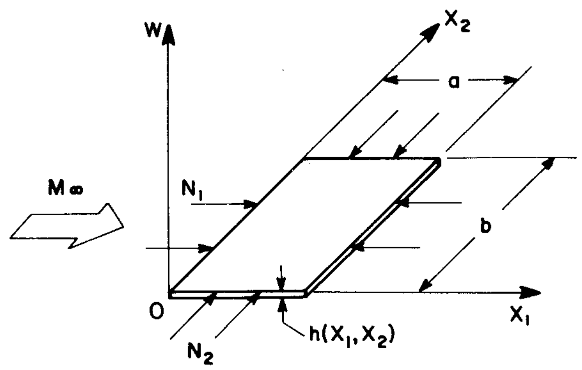

Fig. 1 Orthotropic panel with in-plane loads in supersonic flow.

Two types of boundary conditions will be considered in the present analysis:

1) Simply supported (SS) on the whole contour

$$\bar{w}=0, \quad t^3\bar{w},_{11}=0 \quad \text{along} \quad \bar{x}_1=0,1$$
$$\bar{w}=0, \quad t^3\bar{w},_{22}=0 \quad \text{along} \quad \bar{x}_2=0,1 \tag{4}$$

2) Simply supported (SS) along $\bar{x}_1=0,1$ and clamped (CL) along $\bar{x}_2=0,1$

$$\bar{w}=0, \quad t^3\bar{w},_{11}=0 \quad \text{along} \quad \bar{x}_1=0,1$$
$$\bar{w}=0, \quad \bar{w},_2=0 \quad \text{along} \quad \bar{x}_2=0,1 \tag{5}$$

The Optimal Problem

The aeroelastic optimization problem can be formulated as follows. Find the optimal thickness distribution $t(\bar{x}_1,\bar{x}_2)$ that minimizes the functional

$$J=\int_0^1\int_0^1 t(\bar{x}_1,\bar{x}_2)\mathrm{d}\bar{x}_1\mathrm{d}\bar{x}_2 \tag{6}$$

(which represents the mass ratio MR of the optimal panel) subject to the partial differential constraint [Eq. (3)] with boundary conditions [Eq. (4) or (5)] where the speed parameter Λ is held fixed during the optimization process and equal to the flutter critical parameter $(\Lambda_0)_*$ of the uniform thickness reference panel.

Concerning their characteristics (viz., aspect ratio, orthotropicity, in-plane loads as well as boundary conditions), the nonuniform thickness panel is considered similar to its uniform thickness counterpart.

The problem of finding a stationary value of a functional under constraints expressed as partial differential equations belongs to the theory of optimal control of distributed parameter systems. By extending Armand's approach[9] to this optimization problem with a flutter speed constraint, one obtains a system of 45 state and Euler-Lagrange equations in 45 unknowns (12 state variables) with appropriate boundary conditions. The solution of this problem is obviously a formidable task since the majority of the equations are partial differential equations. However, it was possible to reduce exactly and entirely the Euler-Lagrange equations to two fourth-order partial differential equations involving only the deflection $\bar{w}$, thickness ratio t, and Lagrange multiplier μ_{12}. These equations read

$$\bar{E}_1[t^3(\mu_{12,11}+\nu_2\phi^2\mu_{12,22})],_{11}+\phi^2\bar{E}_2[t^3(\nu_1\mu_{12,11}$$
$$+\phi^2\mu_{12,22})],_{22}+2\bar{E}_1A\phi^2(t^3\mu_{12,12}),_{12}-T_{11}\mu_{12,11}$$
$$-\phi^2T_{22}\mu_{12,22}-(\Lambda_0)_*\mu_{12,1}-\bar{\omega}^2t\mu_{12}=0 \tag{7}$$

$$3t^2[\bar{w}_{,11}(\bar{E}_1\mu_{12,11}+\nu_1\bar{E}_2\phi^2\mu_{12,22})+2\bar{E}_1A\phi^2\mu_{12,12}\bar{w}_{,12}$$
$$+\phi^2\bar{E}_2(\nu_1\mu_{12,11}+\phi^2\mu_{12,22})\bar{w}_{,22}]+\phi^2\bar{E}_2-\bar{\omega}^2\bar{w}\mu_{12}=0 \quad (8)$$

Equations (7) and (8) are to be supplemented by the constraint equation (3) in which Λ is replaced by the appropriate critical flutter speed $(\Lambda_0)_*$. Hence,

$$\bar{E}_1[t^3(\bar{w}_{,11}+\nu_2\phi^2\bar{w}_{,22})]_{,11}+\phi^2\bar{E}_2[t^3(\nu_1\bar{w}_{,11}+\phi^2\bar{w}_{,22})]_{,22}$$
$$+2\bar{E}_1A\phi^2(t^3\bar{w}_{,12})_{,12}-T_{11}\bar{w}_{,11}-\phi^2T_{22}\bar{w}_{,22}$$
$$+(\Lambda_0)_*\bar{w}_{,1}-\bar{\omega}^2t\bar{w}=0 \quad (9)$$

In this way, the optimality conditions reduce to the three simultaneous partial differential equations (7-9) in three dependent variables $\bar{w}(\bar{x}_1,\bar{x}_2)$, $t(\bar{x}_1,\bar{x}_2)$, and $\mu_{12}(\bar{x}_1,\bar{x}_2)$ subjected to the boundary conditions [Eq. (4) or (5)] and to the following additional set pertaining to μ_{12}:

1) Simply supported

$$\mu_{12}=0, \quad t^3\mu_{12,11}=0 \quad \text{along} \quad \bar{x}_1=0,1$$
$$\mu_{12}=0, \quad t^3\mu_{12,22}=0 \quad \text{along} \quad \bar{x}_2=0,1 \quad (10)$$

2) Simply supported along $\bar{x}_1=0,1$ and clamped along $\bar{x}_2=0,1$

$$\mu_{12}=0, \quad t^3\mu_{12,11}=0 \quad \text{along} \quad \bar{x}_1=0,1$$
$$\mu_{12}=0, \quad t^3\mu_{12,2}=0 \quad \text{along} \quad \bar{x}_2=0,1 \quad (11)$$

It may be remarked that Eqs. (7) and (9) differ only by the sign of the odd derivative term. This may be physically interpreted as resulting from a reversal of the flow direction. Equation (7) is referred to as the adjoint of Eq. (9). Since the boundary conditions are the same, as it results by comparing Eqs. (4) and (5) with Eqs. (10) and (11), the eigenvalues for Eqs. (7) and (9) are the same.

As this point it should be remarked that the frequency parameter $\bar{\omega}^2$, which plays the role of an eigenvalue in Eqs. (7) and (9), is as yet undetermined. In the absence of damping, the flutter critical parameters of aeroelastic systems (i.e., velocity and frequency) may be obtained in the $(\bar{\omega}^2,\Lambda)$ plane as the coordinates of the point where two eigenfrequencies are coalescing.[9] Accordingly, the value of $\bar{\omega}^2$ in the optimal solution must be determined so as to constitute, together with the velocity parameter $\Lambda=(\Lambda_0)_*$ ab initio held fixed, the flutter parameters of the minimum weight panel. Due to the adjoint character of Eqs. (7) and (9), the above requirement may be expressed in terms of the Plaut's flutter instability condition[7] in the form

$$\int_0^1\int_0^1 t\bar{w}\mu_{12}\,\mathrm{d}\bar{x}_1\,\mathrm{d}\bar{x}_2=0 \quad (12)$$

The aeroelastic optimization problem reduces at this stage to Eqs. (7-9) and (12) in the four unknowns $t(\bar{x}_1,\bar{x}_2)$, $\bar{w}(\bar{x}_1,\bar{x}_2)$, $\mu_{12}(\bar{x}_1,\bar{x}_2)$, and $(\bar{\omega}^2_{\mathrm{opt}})_*$ (which denotes the flutter frequency parameter of the optimal panel), subject to the boundary conditions [Eqs. (4) and (10) or (5)]. The value $(\bar{\omega}^2_{\mathrm{opt}})_*$ thus obtained is guaranteed to correspond to a minimum weight panel having the same flutter speed as that of the reference panel. Equation (12)—used here for the first time to obtain directly the exact value of $(\bar{\omega}^2_{\mathrm{opt}})_*$—replaces an approximate procedure proposed by Weisshaar[10] and used by Pierson[4] in which both $t(\bar{x}_1,\bar{x}_2)$ and $(\bar{\omega}^2_{\mathrm{opt}})_*$ are varied in seeking the minimum weight panel solution.

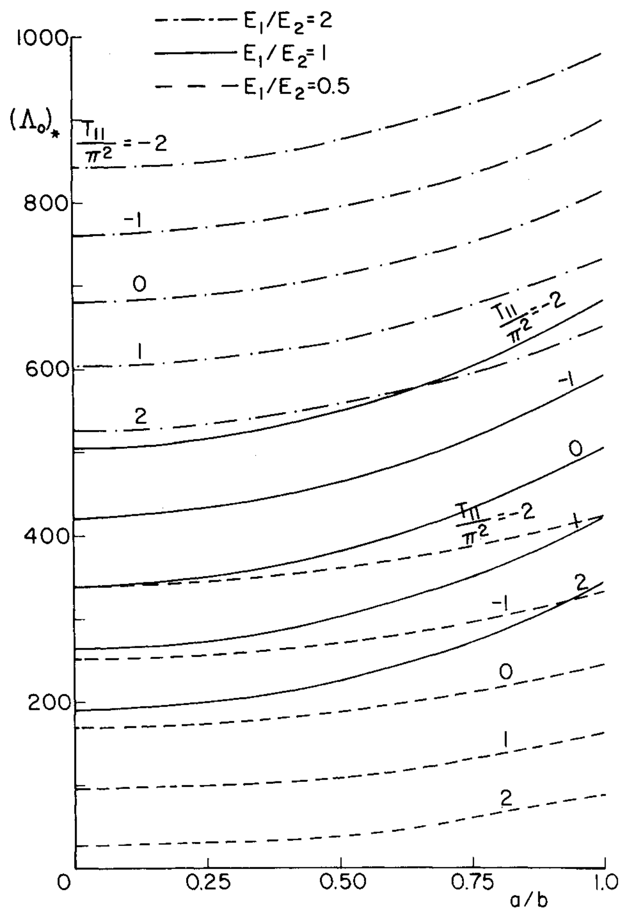

Fig. 2 Flutter speed parameter variation with aspect ratio for uniform panels with all edges SS.

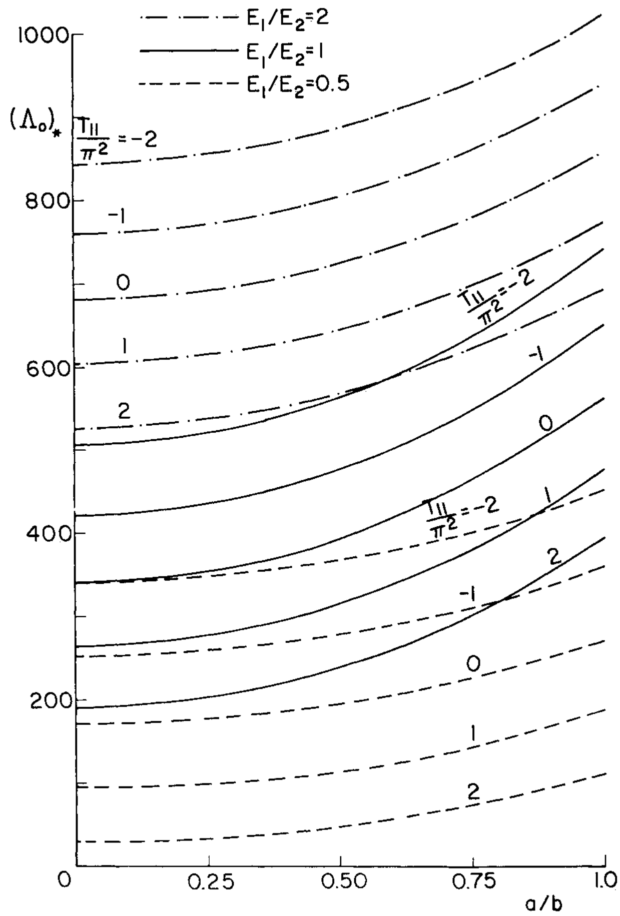

Fig. 3 Flutter speed parameter variation with aspect ratio for uniform panels with spanwise edges SS and streamwise edges CL.

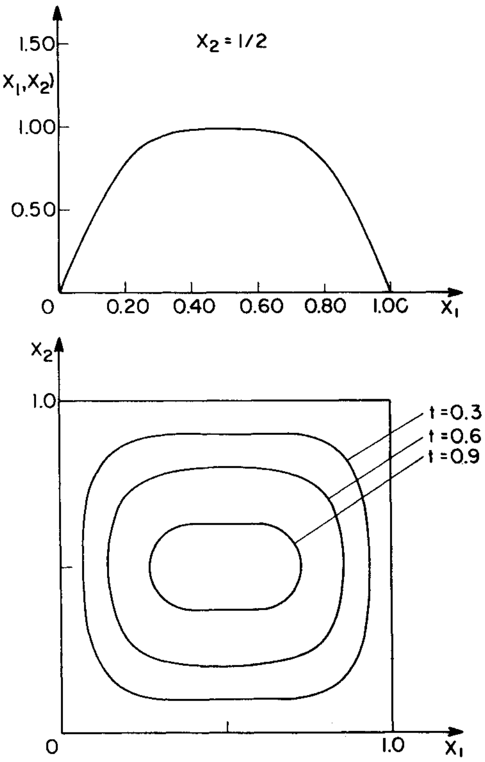

Fig. 4 Typical optimal thickness distributions for orthotropic panels with fixed flutter speed.

Final Form of the Optimality Equations

Let us change $\bar{x}_1$ into $(1-\bar{x}_1)$ and $\bar{x}_2$ into $(1-\bar{x}_2)$ in Eqs. (7-9) and (12) and also in the boundary conditions. By denoting $\tilde{w} \equiv \bar{w}(1-\bar{x}_1, 1-\bar{x}_2)$, $\tilde{\mu}_{12} \equiv \mu_{12}(1-\bar{x}_1, 1-\bar{x}_2)$, and $\tilde{t} \equiv t\ (1-\bar{x}_1, \bar{x}_2)$, one can verify that between the original equations and their transformed counterparts there may be established the following correspondence relations: $\bar{w} \leftrightarrow \tilde{\mu}_{12}$, $\mu_{12} \leftrightarrow \tilde{w}$, and $t \leftrightarrow \tilde{t}$. Hence, a possible solution of the problem can be expressed in the form

$$\mu_{12}(\bar{x}_1, \bar{x}_2) = B\bar{w}(1-\bar{x}_1, 1-\bar{x}_2)$$

or

$$\bar{w}(\bar{x}_1, \bar{x}_2) = \frac{1}{B}\mu_{12}(1-\bar{x}_1, 1-\bar{x}_2) \tag{13}$$

and

$$t(\bar{x}_1, \bar{x}_2) = t(1-\bar{x}_1, 1-\bar{x}_2) \tag{14}$$

Equation (14) expresses the symmetry of the optimal thickness distribution about the panel mid-axes $\bar{x}_1 = \bar{x}_2 = ½$. This result generalizes the thickness symmetry property previously shown to exist in the one-dimensional case.

B is a dimensionless constant that takes on only negative values. For convenience, B is set equal to -1 and Eq. (8) can be rewritten in terms of $t(\bar{x}_1, \bar{x}_2)$, $\bar{w}(\bar{x}_1, \bar{x}_2)$, and $\bar{w}(1-\bar{x}_1, 1-\bar{x}_2)$ $(\equiv \tilde{w})$

$$3t^2 [\tilde{w}_{,11} (\bar{E}_1 \bar{w}_{,11} + \nu_1 \bar{E}_2 \phi^2 \bar{w}_{,22}) + 2\bar{E}_1 A \phi^2 \tilde{w}_{,12} \bar{w}_{,12}$$

$$+ \phi^2 \bar{E}_2 (\nu_1 \bar{w}_{,11} + \phi^2 \bar{w}_{,22}) \tilde{w}_{,22}] + \phi^2 \bar{E}_2 - \bar{\omega}^2 \bar{w}\tilde{w} = 0 \tag{15}$$

The Lagrange multiplier μ_{12} being thus eliminated, the aeroelastic optimization problem is finally reduced to the set of two nonlinear partial differential equations (9) and (15) and the integral condition (12) modified in the form

$$\int_0^1 \int_0^1 t\bar{w}\tilde{w} \mathrm{d}\bar{x}_1 \mathrm{d}\bar{x}_2 = 0 \tag{16}$$

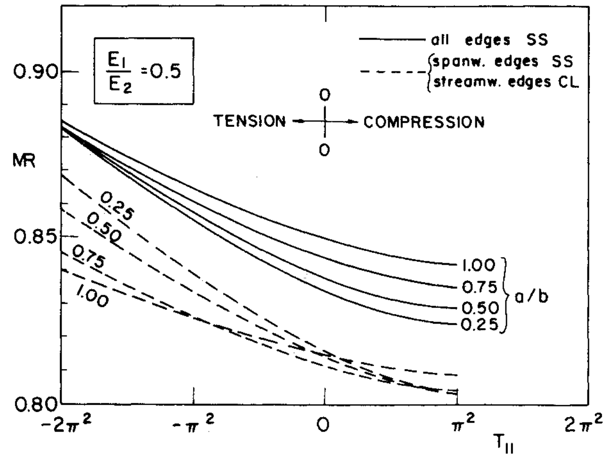

Fig. 5 Dependence of mass ratio on in-plane loading for optimal panels with fixed flutter speed.

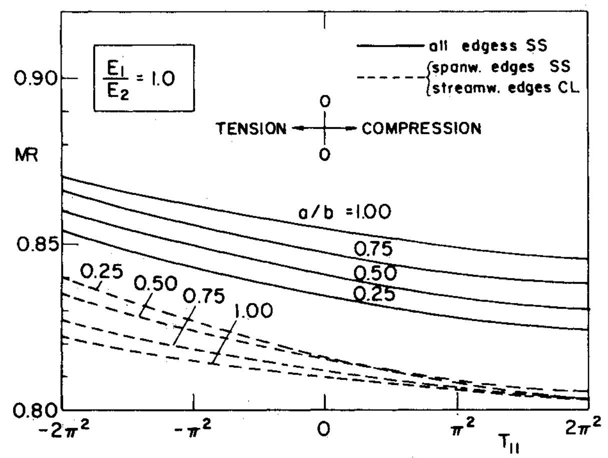

Fig. 6 Dependence of mass ratio on in-plane loading for optimal fixed flutter speed.

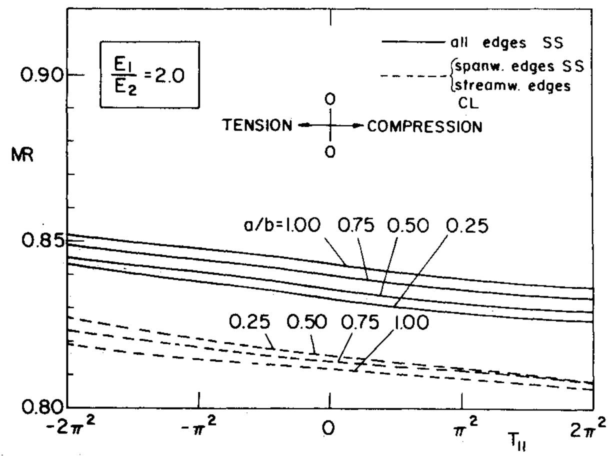

Fig. 7 Dependence of mass ratio on in-plane loading for optimal panels with fixed flutter speed.

involving the unknowns t, $\bar{w}$, and $(\bar{\omega}^2_{\mathrm{opt}})_*$. This set must be solved for given boundary conditions; specified values of the orthotropy ratio E_1/E_2, in-plane loads T_{11} and T_{22}, inverse aspect ratio a/b; and a prescribed value of $(\Lambda_0)_*$ that corresponds to that of the reference panel.

Numerical Results

The resulting nonlinear boundary value problem could be solved by using Pierson's approach[4] based on an approxima-

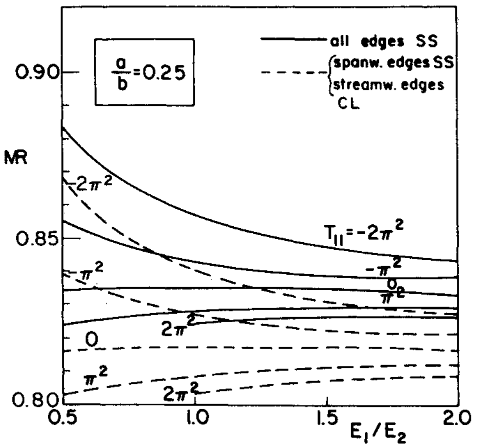

Fig. 8 Depedence of mass ratio on orthotropicity ratio for optimal panels with fixed flutter speed.

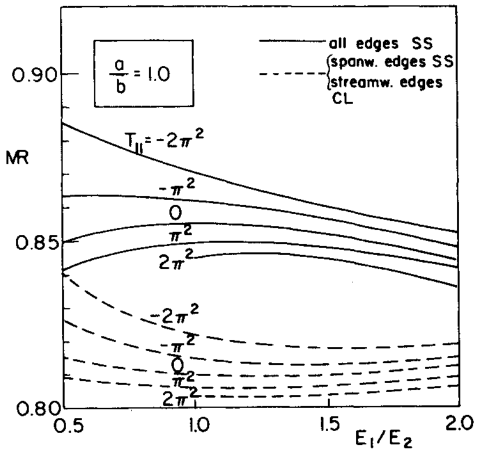

Fig. 9 Dependence of mass ratio on orthotropicity ratio for optimal panels with fixed flutter speed.

tion of the governing partial differential equations and coupled with a gradient projection algorithm. Alternatively, a standard relaxation procedure applied after approximating the partial derivatives with finite differences[3] may be used. However, both methods have been reported to meet with considerable difficulties. In order to obtain just a first estimate of solution, a Galerkin technique[5] was employed here (for the substantiation of Galerkin's method to nonconservative systems, see Ref. 11). To this end, the following trigonometric series representations consistent with the considered boundary conditions and the theoretically proved symmetry with respect to $x_1 = \bar{x}_2 = 1/2$ have been adopted:

1) All edges SS

$$\bar{w}(\bar{x}_1, \bar{x}_2) = (a_1 \sin\pi\bar{x}_1 + a_2 \sin 2\pi\bar{x}_1 + a_3 \sin 3\pi\bar{x}_1)\sin\pi\bar{x}_2$$

$$f(\bar{x}_1, \bar{x}_2) = (b_1 \sin\pi\bar{x}_1 + b_3 \sin 3\pi\bar{x}_1)\sin\pi\bar{x}_2 \tag{17}$$

2) Spanwise edges SS, streamwise edges CL

$$\bar{w}(\bar{x}_1, \bar{x}_2) = (a_1 \sin\pi\bar{x}_1 + a_2 \sin 2\pi\bar{x}_1 + a_3 \sin 3\pi\bar{x}_1)(1 - \cos 2\pi\bar{x}_2)$$

$$f(\bar{x}_1, \bar{x}_2) = (b_1 \sin\pi\bar{x}_1 + b_3 \sin 3\pi\bar{x}_1)\sin\pi\bar{x}_2 \tag{18}$$

It should be noted that the deflection function $\bar{w}(\bar{x}_1, \bar{x}_2)$ was represented as a summation of terms only in the x_1 direction.

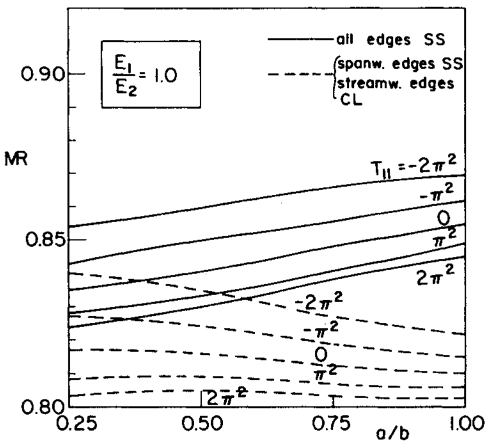

Fig. 10 Dependence of mass ratio on aspect ratio for optimal panels with fixed flutter speed.

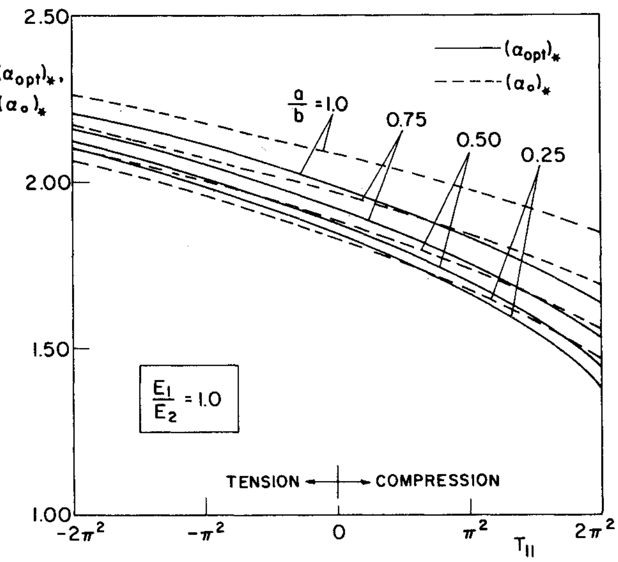

Fig. 11 Flutter frequency parameter variation with in-plane loading for panels with all edges SS.

The justification lies in the fact that the expression of aerodynamic loads entering the aeroelastic equilibrium equations involve only $\bar{x}_1$ derivatives of $\bar{w}$ (a fact that characterizes both the exact and approximate two- and three-dimensional aerodynamic theories. This mathematical feature corroborated with the fact that these loads are ultimately responsible for the appearance of the flutter phenomenon suggest that representations [Eqs. (17) and (18)] for $\bar{w}$ are appropriate for flutter calculations and implicitly for the weight minimization of panels subject to aeroelastic constraints.

Substitution of Eq. (17) or (18) into Eqs. (9), (15), and (16) and employment of the standard Galerkin procedure yields in each case a set of six nonlinear algebraic equations in terms of the unknown coefficients a_1, a_2, a_3, b_1, b_3, and $(\bar{\omega}^2_{opt})_*$. This set of equations was solved using a Newton-Raphson algorithm.

Values of $(\Lambda_0)_*$ have been computed via Galerkin's procedure for the following ranges of the relevant parameters:

$$a/b = 0.25, \quad 0.50, \quad 0.75, \quad 1.00$$

$$T_{11} = -2\pi^2, \quad -\pi^2, \quad 0, \quad \pi^2, \quad 2\pi^2$$

$$E_1/E_2 = 0.5, \quad 1.0, \quad 2.0 \tag{19}$$

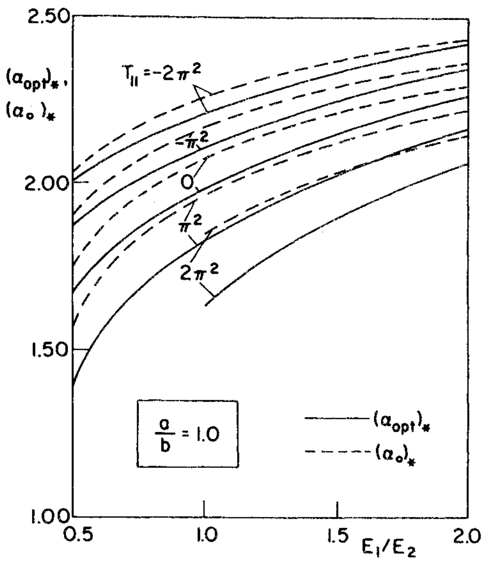

Fig. 12 Flutter frequency parameter variation with orthotropicity ratio for panels with all edges SS.

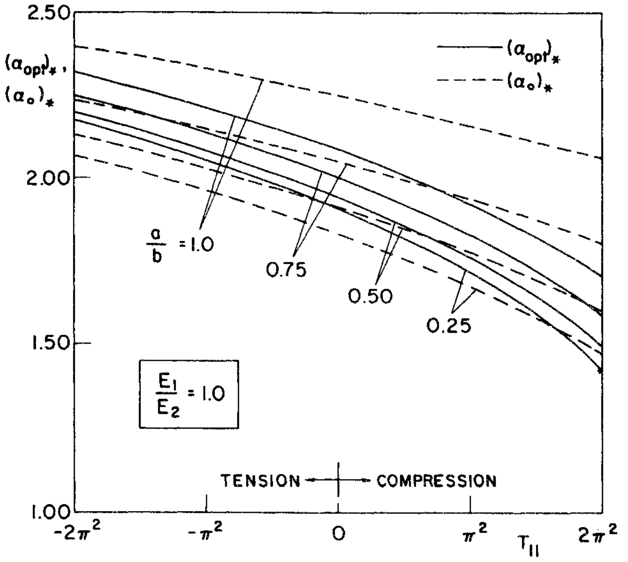

Fig. 13 Flutter frequency parameter variation with in-plane loading for panels with spanwise edges SS and streamwise edges CL.

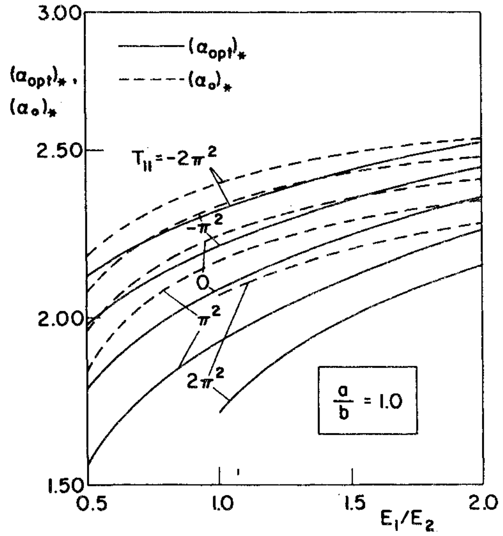

Fig. 14 Flutter frequency parameter variation with orthotropy for panels with spanwise edges SS and streamwise edges CL.

and for both support equations (17) and (18).[12] The critical flutter speed in all cases was found to be independent of T_{22}, a result similar to that obtained by Hedgepeth for isotropic SS panels.[13] The dependence of $(\Lambda_0)_*$ on the various parameters is presented in Figs. 2 and 3.

The parameter values exhibited in Eq. (19) together with the corresponding flutter speeds $(\Lambda_0)_*$ given in Figs. 2 and 3 have been used to obtain optimal panel designs by solving the system of nonlinear algebraic equations in the unknowns a_1, a_2, b_1, b_3, and $(\bar{\omega}^2_{opt})_*$. This was done using the IMSL subroutine ZSPOW on a Cyber 205 computer. A typical optimal thickness distribution is displayed in Fig. 4, showing a profile quite similar to that obtained in Ref. 5, also via Galerkin's method for an isotropic square panel without in-plane loads. A significant outcome of the calculations is the virtual independence of the mass ratio of optimal panels upon T_{22} as it was also found when calculating $(\Lambda_0)_*$ of the reference panels. The dependence of the mass ratio on in-plane loading for various a/b and E_1/E_2 in both support conditions is depicted in Figs. 5-7. The dominant trend is the increase in the mass ratio (that is, a decrease of the weight saving) as increased tension is applied, a conclusion valid for both considered support conditions. The same figures also reveal that, for a given set of parameters, a greater weight reduction is possible for the clamped panel (a similar trend has been obtained by Pierson[14] for the infinite span panel). The influence of the orthotropy on mass ratio is illustrated in Figs. 8 and 9. This influence appears to be coupled to the in-plane loading in the sense that the mass ratio decreases with the orthotropy ratio for tensile loads and increases for compressive loads, a trend valid for both cases of boundary conditions. With regard to the influence of the inverse aspect ratio, Figs. 5-7 and 10 show that the mass ratio increases slightly with a/b for SS panel, while for the CL panel it is virtually independent of a/b.

Figures 11-14 depict the flutter frequency variation with in-plane loading and orthotropicity ratio in both considered support conditions for the optimal panel and its uniform thickness counterpart, where $(\bar{\omega}^2_{opt})_* = (\pi\alpha_{opt})^4_*$ and $(\bar{\omega}^2_0)_* = (\pi\alpha_0)^4_*$. As it can be seen, $(\alpha_{opt})_*$ values tend to remain close to and be higher than the corresponding $(\alpha_0)_*$ values, especially for tensile stresses and lower a/b. Also, higher $(\alpha_{opt})_*$ are obtained for the clamped panel, which is physically consistent with the lower mass ratio of those panels as compared to their simply supported counterparts. These results agree with those obtained by Pierson[14] in the one-dimensional case. Finally, it should be noted that both critical flutter parameters $(\Lambda_0)_*$ and α_* of the optimal and reference panel exhibit similar patterns of variation, by increasing with tensile in-plane loadings, orthotropy ratio, and inverse aspect ratio.

It should be remarked that the Galerkin-type optimal solutions presented above exhibit a zero thickness distribution along the panel edges. However, it must be stressed that theoretical framework developed in the paper can accommodate a minimum thickness constraint which could follow the lines indicated in Refs. 11 and 14.

Conclusions

A theoretical framework for the weight minimization problem of orthotropic, rectangular, supersonic panels subjected to a flutter speed constraint and to a system of uniform in-plane loadings has been developed.

The pertinent optimality equations are obtained using the theory of optimal control of distributed parameter systems. These equations were supplemented by the Plaut instability condition that allows exact determination of the optimal flutter frequency parameter, thus becoming a part of the necessary optimality conditions.

Numerical estimates of the optimal solution were obtained via Galerkin's method. In spite of the approximate nature of this analysis, it was possible to draw a number of qualitative conclusions, as follows:

1) The mass ratio of optimal panels is virtually independent of the spanwise in-plane loads.

2) Larger streamwise tensile in-plane loads result in lower weight savings for both considered boundary conditions.

3) The clamping of streamwise edges results in higher weight savings.

4) Weight saving increases with orthotropy ratio for tensile in-plane loads and decreases for compressive loads for both support conditions.

5) Weight saving decreases slightly with inverse aspect ratio for the simply supported panel, while for the clamped panel the dependence on this parameter is almost insignificant.

6) Flutter frequency parameter values of the optimal panel are generally higher than those of the reference panel, being larger in the case of clamped panels.

7) Critical flutter parameters (speed and frequency) for both optimal and reference panels vary in the same manner, i.e., increase with orthotropy ratio, tensile in-plane loads, and inverse aspect ratio.

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